Theory and Practice of Chunked Sequences

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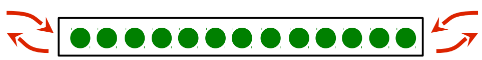
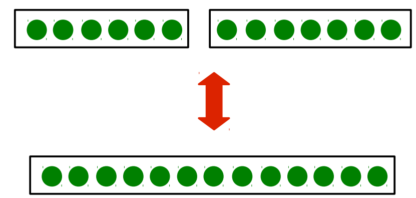
Goal

Ephemeral deques (double-ended queues) with:

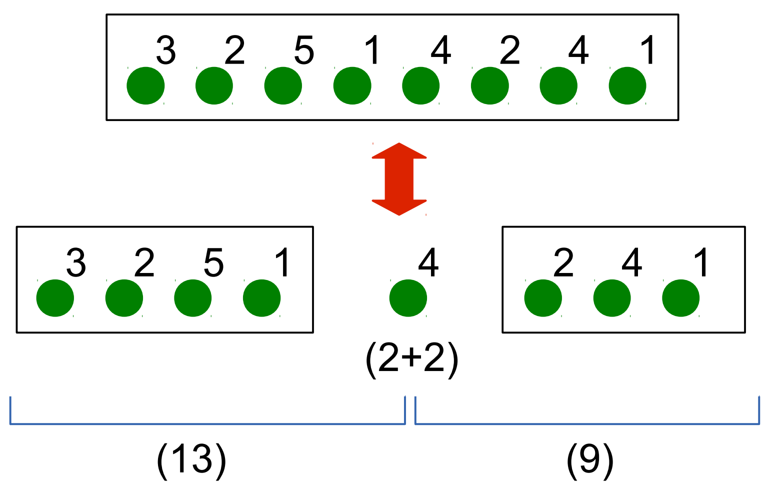
→ push and pop at the two ends, in O(1) amortized

→ concat, and split at arbitrary indices, in O(log n)





Weighted split operation: (e.g. split at 13)



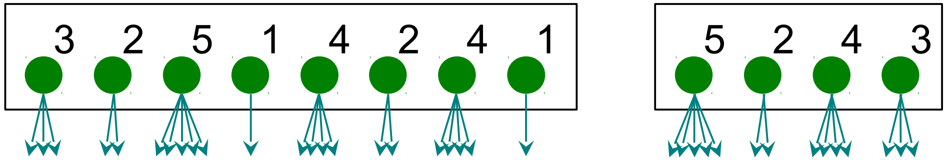
Motivation

Application to parallel BFS and parallel DFS

→ each processor push/pop vertices from its frontier

→ split is used for dynamically distributing the load

→ concatenation is used to merge the new frontiers



Requirements:

→ push and pop must be very efficient

→ split and concatenation is sublinear time

Contribution

Amortized O(1) push/pop and O(log N) concat/split

Prior work:

→ Kaplan and Tarjan (1996)
→ Hinze and Parterson (2006)

Purely functional data structures (confluently persistent).
Yet, very large constant factors (even if made ephemeral).

This work: ephemeral catenable/splittable deques with small constant factors (e.g., not far from C++ STL deques).

Overview

(1) Chunked sequences

→ assume a potentially-slow deque data structure

→ construct a fast deque data structure, using chunks

- amortize allocations in worst-case push-pop scenarios
- ensure space efficiency in worst-case concat scenarios

(2) Bootstrapped chunked sequences

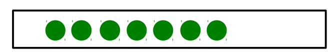
→ build a stand-alone, fast, catenable/splittable deque

→ use structural decomposition and recursive slowdown

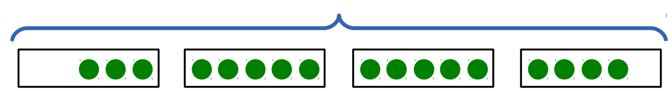
(Dietz 1982; Buchsbaum & Tarjan 1995; Kaplan & Tarjan 1996)

Challenges with chunks

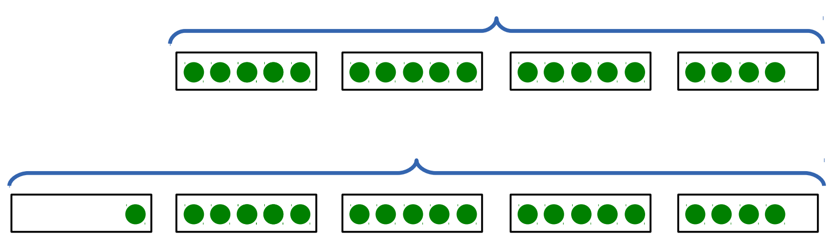
A chunk = fixed-capacity ring buffer (repr. as an array)



A deque of chunks (e.g. C++ STL deques)

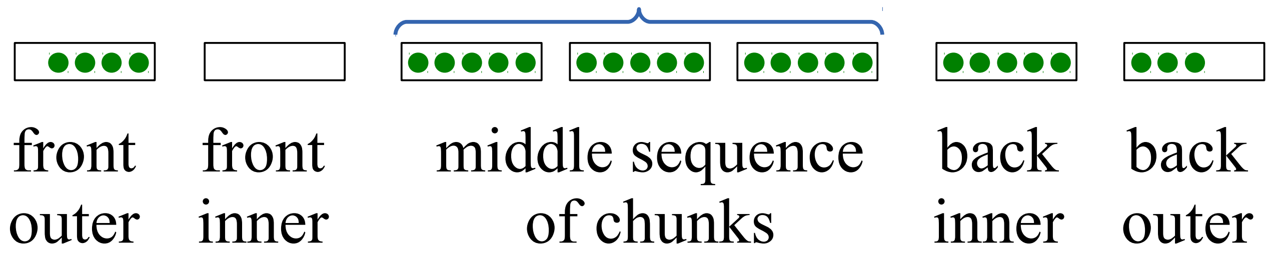


Challenge: (iterated push/pop operations)



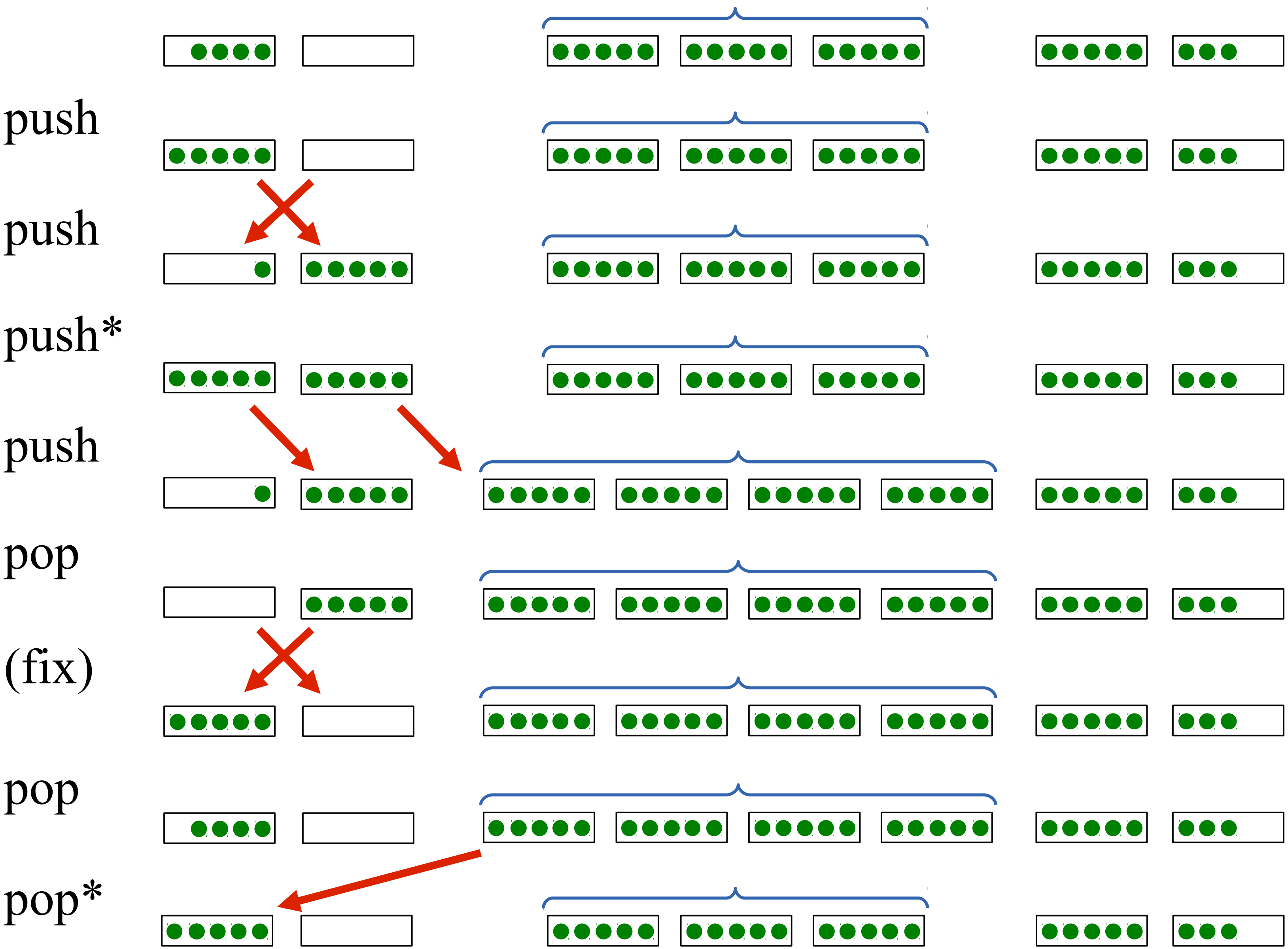
Our chunked sequence

Approach: place two special chunks on each side



Invariant: the inner chunks must be either empty or full.

Implementation of push and pop



Amortization with chunked sequences

Theorem: the amortized cost of push (including pop) is

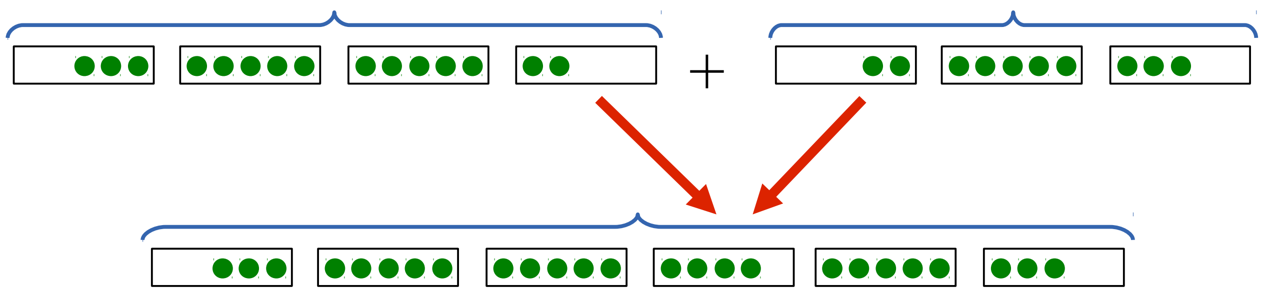
$$C + \frac{A+M}{K} + O(1)$$

where:

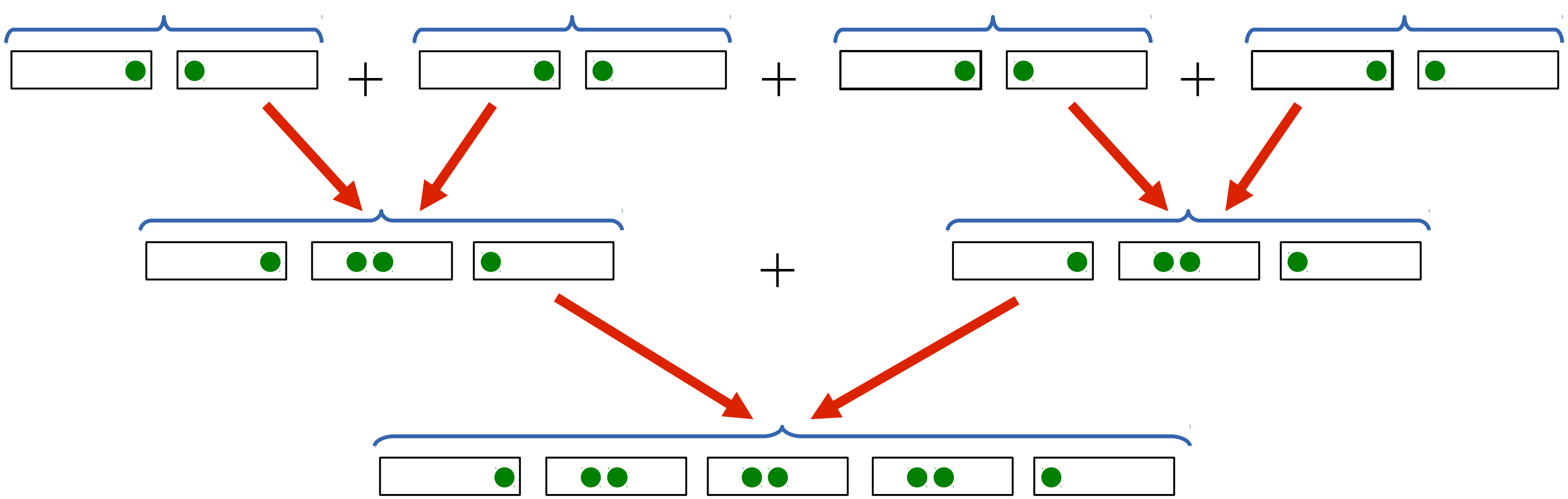
- $\boldsymbol{C}$ cost of push (including associated pop) in a chunk
- $\boldsymbol{A}$ cost of allocation (including deallocation) of a chunk
- $\boldsymbol{M}$ cost of push (including pop) in the middle sequence
- $\boldsymbol{K}$ capacity of a chunk

Challenges with concatenation

Concatenation of deques of chunks (with merge)

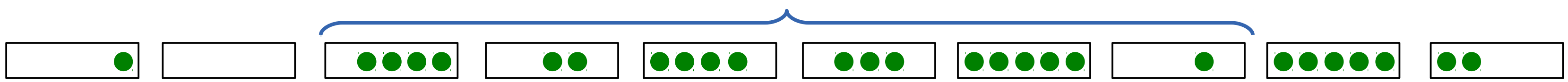


Worst-case scenario:

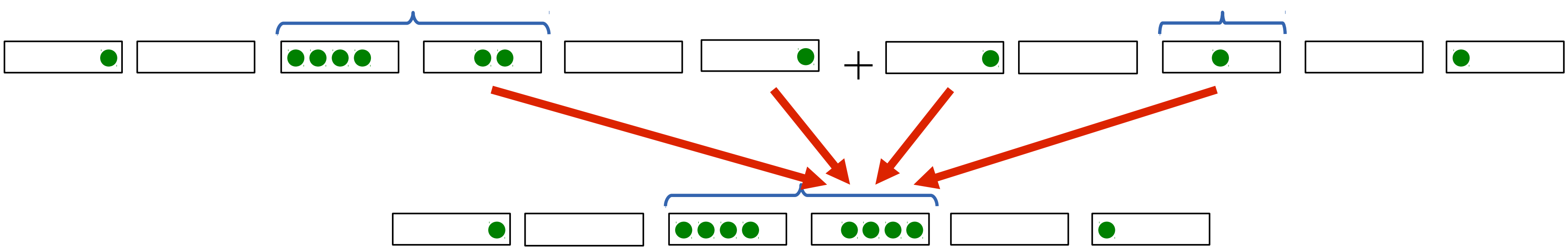


Implementation of concatenation

Invariant: any two consecutive chunks in the middle sequence must store a total number of more than K items.

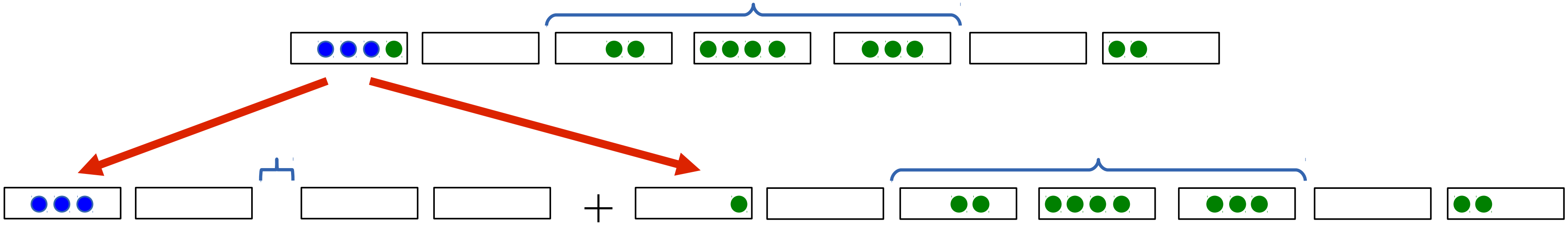


Concatenation: (up to 4 chunks need to be merged)

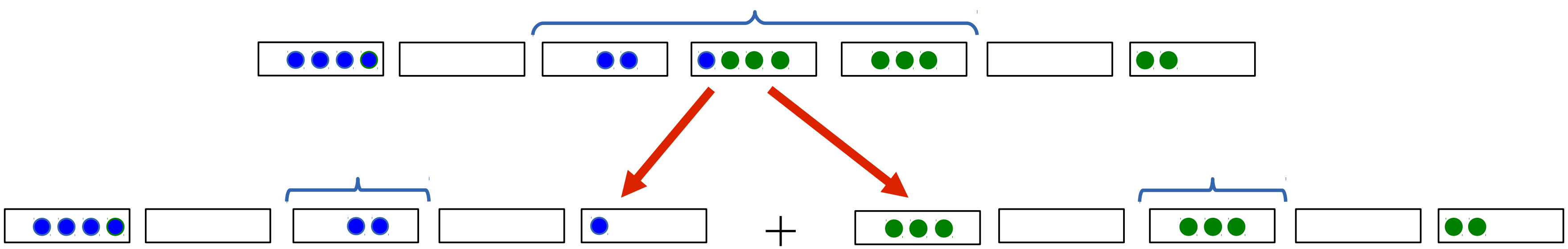


Implementation of split

Case 1:

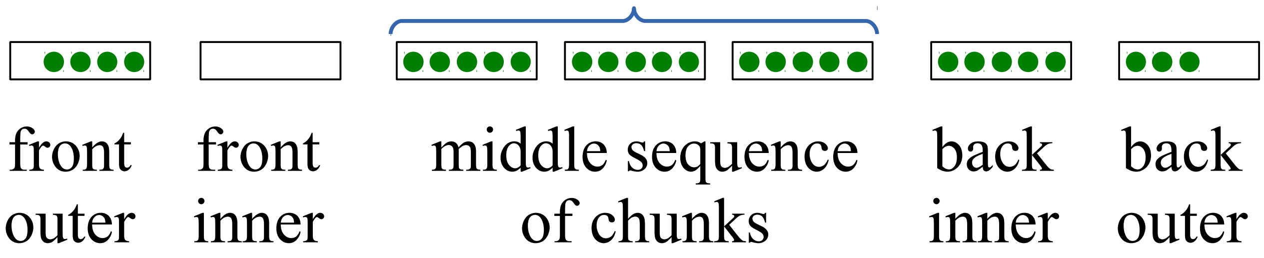


Case 2:



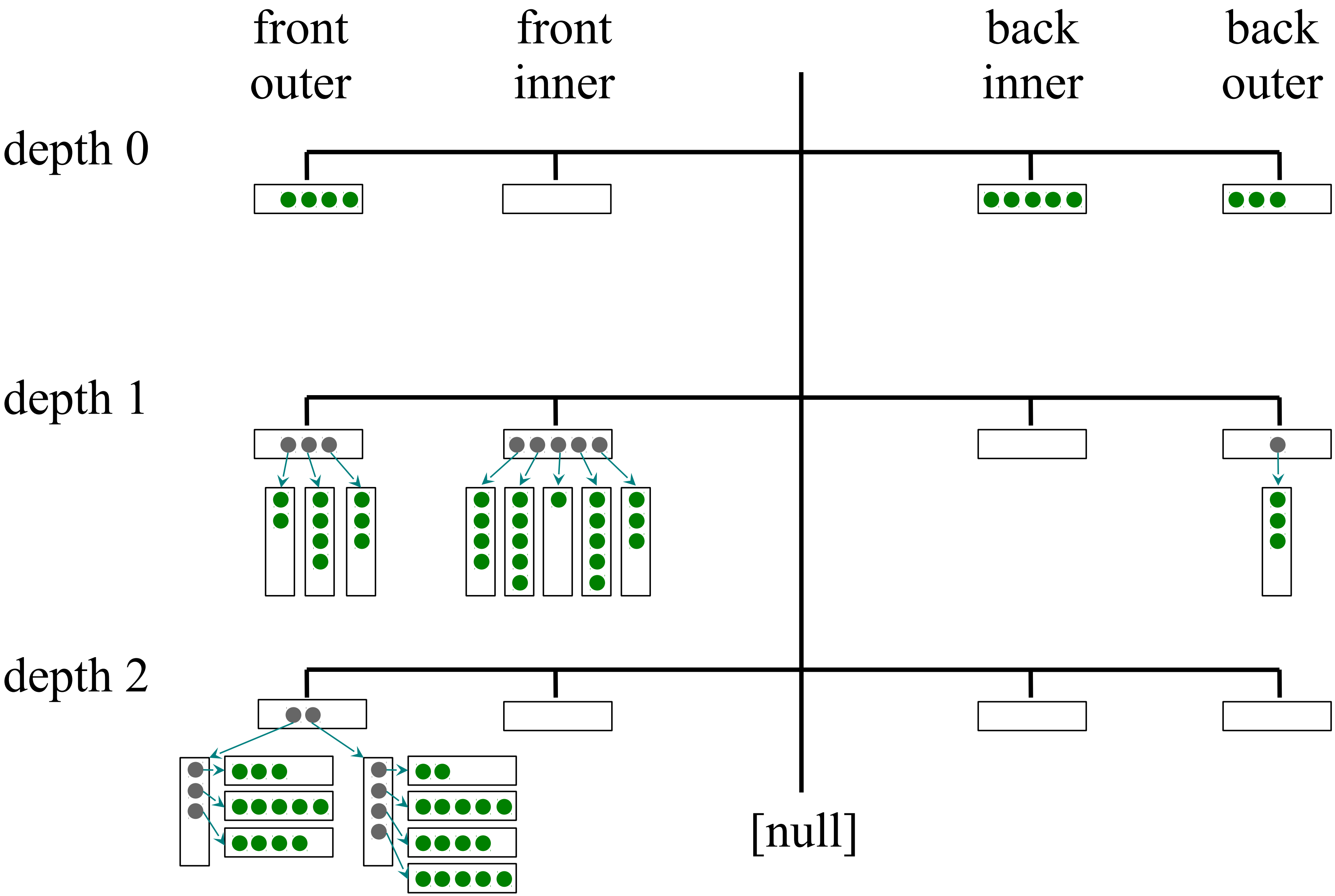
Towards bootstrapping

Summary: given a potentially-slow catenable/splittable deque, we built a catenable/splittable deque structure with small constant factors, even in worst-case scenarios.



Next step: implement the middle sequence of our chunked sequence using... our chunked sequence. Do so recursively.

Bootstraped chunked sequence



Efficiency analysis

Theorem: the depth is at most

$$\left\lfloor \log_{(K+1)/2} n \right\rfloor + 1$$

Remark: depth is bounded by 7 for all practical purposes.

Theorem: push/pop has cost O(1), with a small constant

Theorem: concat and split have cost

$$O\left(K \ * \ \log_{(K+1)/2}\left(\min\left(n_1, n_2\right)\right)\right)$$

where n_1 and n_2 denote the size of the two parts involved.

$\rightarrow$ compare with: $O\left(\log_2\left(\min\left(n_1, n_2\right)\right)\right)$

Space-usage analysis

Theorem: asymptotic space usage is

$$\left(2 + \frac{O(1)}{K}\right) * n$$

Alternative: with concat twice slower, density is 3/4, thus

$$\left(1.33 + \frac{O(1)}{K}\right) * n$$

Alternative: for bag semantics (unordered items), usage is

$$\left(1 + \frac{O(1)}{K}\right) * n$$

Implementation and benchmarks

Two implementations:

→ OCaml (mechanized proof using CFML, in Coq)

→ C++ (carefully optimized code)

Performance of C++ code:

→ first layer with unweighted chunks of capacity 512

→ deeper layers with weighted chunks of capacity 32

Experiment	STL deque	Bootstr. chunked
LIFO ($10^6 * 10^3$)	5.46	+28%
LIFO ($10^3 * 10^6$)	9.15	+20%
LIFO ($10^0 * 10^9$)	12.07	+12%
FIFO ($10^6 * 10^3$)	5.51	+16%
FIFO ($10^3 * 10^6$)	9.16	+15%
FIFO ($10^0 * 10^9$)	12.32	+8%

Thanks!