Pretty-big-step semantics

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Introduction

Operational semantics fall in two categories: small-step and big-step.

Big-step semantics suffer from a serious duplication problem.

Pretty-big-step semantics solve this duplication problem.

Why care about big-step semantics?

	Papers with	Papers with	
	big-step semantics	small-step semantics	
ICFP'11	5	3	
POPL'11	7	16	
ICFP'12	5	4	

Big-step semantics are useful.

Content of this talk

- Ouplication associated with big-step semantics
- From big-step to pretty-big-step semantics
- Scaling up to real languages

Duplication associated with big-step semantics

Big-step semantics for loops: regular behavior

Semantics of a C-style loop "for (; t_1 ; t_2) { t_3 }", written "for t_1 t_2 t_3 ", in terms of the evaluation judgment $t_{/m} \Rightarrow v_{/m'}$.

$$\frac{t_{1/m_1} \Rightarrow \mathsf{false}_{/m_2}}{\mathsf{for}\, t_1\, t_2\, t_{3/m_1} \Rightarrow t t_{/m_2}}$$

$$\frac{t_{1/m_1} \Rightarrow \mathsf{true}_{/m_2} \quad t_{3/m_2} \Rightarrow tt_{/m_3} \quad t_{2/m_3} \Rightarrow tt_{/m_4} \quad \mathsf{for} \, t_1 \, t_2 \, t_{3/m_4} \Rightarrow tt_{/m_5}}{\mathsf{for} \, t_1 \, t_2 \, t_{3/m_1} \Rightarrow tt_{/m_5}}$$

Big-step semantics for loops: exceptions

Exceptions in terms of the judgment $t_{/m} \Rightarrow^{\text{exn}}_{/m'}$.

$$\frac{t_{1/m_1} \Rightarrow^{\text{exn}}_{/m_2}}{\text{for } t_1 \, t_2 \, t_{3/m_1} \Rightarrow^{\text{exn}}_{/m_2}}$$

$$\frac{t_{1/m_1} \Rightarrow \text{true}_{/m_2} \quad t_{3/m_2} \Rightarrow^{\text{exn}}_{/m_3}}{\text{for } t_1 \, t_2 \, t_{3/m_1} \Rightarrow^{\text{exn}}_{/m_3}}$$

$$\frac{t_{1/m_1} \Rightarrow \text{true}_{/m_2} \quad t_{3/m_2} \Rightarrow tt_{/m_3} \quad t_{2/m_3} \Rightarrow^{\text{exn}}_{/m_4}}{\text{for } t_1 \, t_2 \, t_{3/m_1} \Rightarrow^{\text{exn}}_{/m_4}}$$

$$\frac{t_{1/m_1} \Rightarrow \text{true}_{/m_2} \quad t_{3/m_2} \Rightarrow tt_{/m_3} \quad t_{2/m_3} \Rightarrow tt_{/m_4} \quad \text{for } t_1 \, t_2 \, t_{3/m_4} \Rightarrow^{\text{exn}}_{/m_5}}{\text{for } t_1 \, t_2 \, t_{3/m_1} \Rightarrow^{\text{exn}}_{/m_5}}$$

Big-step semantics for loops: divergence

Divergence in terms of the coinductive judgment $t_{/m} \Rightarrow^{\infty}$ (Leroy 2006).

$$\begin{split} \frac{t_{1/m_1} \Rightarrow^\infty}{\text{for } t_1 \, t_2 \, t_{3/m_1} \Rightarrow^\infty} \\ \frac{t_{1/m_1} \Rightarrow \text{true}_{/m_2} \quad t_{3/m_2} \Rightarrow^\infty}{\text{for } t_1 \, t_2 \, t_{3/m_1} \Rightarrow^\infty} \\ \frac{t_{1/m_1} \Rightarrow \text{true}_{/m_2} \quad t_{3/m_2} \Rightarrow t_{1/m_3} \quad t_{2/m_3} \Rightarrow^\infty}{\text{for } t_1 \, t_2 \, t_{3/m_1} \Rightarrow^\infty} \\ \frac{t_{1/m_1} \Rightarrow \text{true}_{/m_2} \quad t_{3/m_2} \Rightarrow t_{1/m_3} \quad t_{2/m_3} \Rightarrow t_{1/m_4} \quad \text{for } t_1 \, t_2 \, t_{3/m_4} \Rightarrow^\infty}{\text{for } t_1 \, t_2 \, t_{3/m_1} \Rightarrow^\infty} \end{split}$$

Big-step semantics for loops: summary

$$\frac{t_{1/m_1} \,\Rightarrow\, \mathsf{false}_{/m_2}}{\mathsf{for}\, t_1\, t_2\, t_{3/m_1} \,\Rightarrow\, t\!t_{/m_2}}$$

$$\frac{t_{1/m_1} \Rightarrow \mathsf{true}_{/m_2} \quad t_{3/m_2} \Rightarrow t_{1/m_3} \quad t_{2/m_3} \Rightarrow t_{1/m_4} \quad \mathsf{for} \, t_1 \, t_2 \, t_{3/m_4} \Rightarrow t_{1/m_5}}{\mathsf{for} \, t_1 \, t_2 \, t_{3/m_1} \Rightarrow t_{1/m_5}}$$

$$\frac{t_{1/m_1} \Rightarrow^{\operatorname{exn}}_{/m_2}}{\operatorname{for} t_1 \ t_2 \ t_{3/m_1} \Rightarrow^{\operatorname{exn}}_{/m_2}}$$

$$\frac{t_{1/m_1} \Rightarrow \mathsf{true}_{/m_2} \quad t_{3/m_2} \Rightarrow^{\mathsf{exn}}_{/m_3}}{\mathsf{for} \, t_1 \, t_2 \, t_{3/m_1} \Rightarrow^{\mathsf{exn}}_{/m_3}}$$

$$\frac{t_{1/m_1} \Rightarrow \operatorname{true}_{/m_2} \ t_{3/m_2} \Rightarrow t_{1/m_3}}{t_{2/m_3} \Rightarrow^{\operatorname{exn}}_{m_4}} \frac{t_{1/m_3}}{\operatorname{for} \ t_1 \ t_2 \ t_{3/m_1} \Rightarrow^{\operatorname{exn}}_{m_4}}$$

$$\frac{t_{1/m_{1}} \Rightarrow \operatorname{true}_{/m_{2}} \ t_{3/m_{2}} \Rightarrow t_{/m_{3}}}{t_{2/m_{3}} \Rightarrow t_{/m_{4}} \ \operatorname{for} t_{1} \ t_{2} \ t_{3/m_{4}} \Rightarrow^{\operatorname{exn}}_{/m_{5}}} \\ \frac{t_{2/m_{3}} \Rightarrow t_{/m_{4}} \ \operatorname{for} t_{1} \ t_{2} \ t_{3/m_{1}} \Rightarrow^{\operatorname{exn}}_{/m_{5}}}{\operatorname{for} t_{1} \ t_{2} \ t_{3/m_{1}} \Rightarrow^{\operatorname{exn}}_{/m_{5}}}$$

$$\frac{t_{1/m_1} \Rightarrow^{\infty}}{\operatorname{for} t_1 \, t_2 \, t_{3/m_1} \Rightarrow^{\infty}}$$

$$\frac{t_{1/m_1} \Rightarrow \mathsf{true}_{/m_2} \quad t_{3/m_2} \Rightarrow^{\infty}}{\mathsf{for}\, t_1\, t_2\, t_{3/m_1} \Rightarrow^{\infty}}$$

$$\frac{t_{1/m_1} \Rightarrow \mathsf{true}/m_2}{t_{2/m_3} \Rightarrow^{\infty}} \xrightarrow{t_{1/m_3}} \frac{t_{1/m_2} \Rightarrow t_{1/m_3}}{\mathsf{for}\, t_1\, t_2\, t_{3/m_1} \Rightarrow^{\infty}}$$

$$\begin{array}{c} t_{1/m_1} \Rightarrow \mathsf{true}_{/m_2} \quad t_{3/m_2} \Rightarrow t_{/m_3} \\ \underline{t_{2/m_3}} \Rightarrow t_{/m_4} \quad \mathsf{for} \, t_1 \, t_2 \, t_{3/m_4} \Rightarrow \\ \hline \mathsf{for} \, t_1 \, t_2 \, t_{3/m_1} \Rightarrow^{\infty} \end{array}$$

Big-step semantics for loops: summary

$$\frac{t_{1/m_{1}} \Rightarrow \mathsf{false}/m_{2}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow tt/m_{2}}$$

$$\frac{t_{1/m_{1}} \Rightarrow \mathsf{true}/m_{2}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{2}} \Rightarrow tt/m_{3}} \xrightarrow{t_{2/m_{3}}} \Rightarrow tt/m_{4}} \quad \mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{4}} \Rightarrow tt/m_{5}}$$

$$\frac{t_{1/m_{1}} \Rightarrow^{\mathsf{exn}}/m_{2}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow^{\mathsf{exn}}/m_{2}} \qquad \qquad \frac{t_{1/m_{1}} \Rightarrow^{\mathsf{exn}}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow^{\mathsf{exn}}/m_{3}}$$

$$\frac{t_{1/m_{1}} \Rightarrow \mathsf{true}/m_{2}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow^{\mathsf{exn}}/m_{3}} \qquad \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{true}/m_{2}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow^{\mathsf{exn}}/m_{3}}$$

$$\frac{t_{1/m_{1}} \Rightarrow \mathsf{true}/m_{2}}{\mathsf{true}/m_{2}} \xrightarrow{\mathsf{exn}}/m_{4} \qquad \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{true}/m_{2}}{\mathsf{true}/m_{2}} \xrightarrow{\mathsf{exn}}/m_{4}} \qquad \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{true}/m_{2}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow^{\mathsf{exn}}/m_{4}} \qquad \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{true}/m_{2}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow^{\mathsf{exn}}/m_{3}} \qquad \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{true}/m_{2}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow^{\mathsf{exn}}/m_{3}} \qquad \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{true}/m_{2}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow^{\mathsf{exn}}/m_{3}} \qquad \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{true}/m_{2}}{\mathsf{true}/m_{2}} \xrightarrow{\mathsf{exn}/m_{3}} \qquad \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{true}/m_{2}}{\mathsf{exn}} \xrightarrow{\mathsf{exn}/m_{3}} \qquad \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{true}/m_{2}}{\mathsf{exn}} \xrightarrow{\mathsf{exn}/m_{3}} \qquad \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{true}/m_{2}}{\mathsf{exn}} \xrightarrow{\mathsf{exn}/m_{3}} \xrightarrow{\mathsf{exn}/m_{3}} \qquad \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{exn}/m_{3}}{\mathsf{exn}} \xrightarrow{\mathsf{exn}/m_{3}} \qquad \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{exn}/m_{3$$

 \rightarrow Even with factorization: 9 rules, 21 premises.

Big-step semantics for loops: summary

$$\frac{t_{1/m_{1}} \Rightarrow \mathsf{false}_{/m_{2}}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow tt_{/m_{2}}}$$

$$\frac{t_{1/m_{1}} \Rightarrow \mathsf{true}_{/m_{2}} \quad t_{3/m_{2}} \Rightarrow tt_{/m_{3}} \quad t_{2/m_{3}} \Rightarrow tt_{/m_{4}} \quad \mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{4}} \Rightarrow tt_{/m_{5}}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow \mathsf{true}_{/m_{2}}}$$

$$\frac{t_{1/m_{1}} \Rightarrow^{\mathsf{exn}}_{/m_{2}}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow^{\mathsf{exn}}_{/m_{2}}} \qquad \frac{t_{1/m_{1}} \Rightarrow^{\mathsf{exn}}_{/m_{2}}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow^{\mathsf{exn}}_{/m_{3}}}$$

$$\frac{t_{1/m_{1}} \Rightarrow \mathsf{true}_{/m_{2}} \quad t_{3/m_{2}} \Rightarrow^{\mathsf{exn}}_{/m_{3}}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow^{\mathsf{exn}}_{/m_{3}}} \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{true}_{/m_{2}} \quad t_{3/m_{2}} \Rightarrow^{\mathsf{exn}}_{/m_{3}}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow^{\mathsf{exn}}_{/m_{4}}} \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{true}_{/m_{2}} \quad t_{3/m_{2}} \Rightarrow^{\mathsf{et}/m_{3}}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow^{\mathsf{exn}}_{/m_{4}}}$$

$$\frac{t_{1/m_{1}} \Rightarrow \mathsf{true}_{/m_{2}} \quad t_{3/m_{2}} \Rightarrow^{\mathsf{et}/m_{3}}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow^{\mathsf{exn}}_{/m_{4}}} \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{true}_{/m_{2}} \quad t_{3/m_{2}} \Rightarrow^{\mathsf{et}/m_{3}}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{1}} \Rightarrow^{\mathsf{exn}}_{/m_{4}}}$$

$$\frac{t_{1/m_{1}} \Rightarrow \mathsf{true}_{/m_{2}} \quad t_{3/m_{2}} \Rightarrow^{\mathsf{et}/m_{3}}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{4}} \Rightarrow^{\mathsf{et}/m_{3}}} \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{true}_{/m_{2}} \quad t_{3/m_{2}} \Rightarrow^{\mathsf{et}/m_{3}}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{4}} \Rightarrow^{\mathsf{et}/m_{3}}}}$$

$$\frac{t_{1/m_{1}} \Rightarrow \mathsf{true}_{/m_{2}} \quad t_{3/m_{2}} \Rightarrow^{\mathsf{et}/m_{3}}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{4}} \Rightarrow^{\mathsf{et}/m_{3}}} \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{true}_{/m_{2}} \quad t_{3/m_{2}} \Rightarrow^{\mathsf{et}/m_{3}}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{4}} \Rightarrow^{\mathsf{et}/m_{3}}}}$$

$$\frac{t_{1/m_{1}} \Rightarrow \mathsf{true}_{/m_{2}} \quad t_{3/m_{2}} \Rightarrow^{\mathsf{et}/m_{3}}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{4}} \Rightarrow^{\mathsf{et}/m_{3}}} \qquad \frac{t_{1/m_{1}} \Rightarrow \mathsf{true}_{/m_{2}} \quad t_{3/m_{2}} \Rightarrow^{\mathsf{et}/m_{3}}}{\mathsf{for}\,t_{1}\,t_{2}\,t_{3/m_{4}} \Rightarrow^{\mathsf{et}/m_{3}}}}$$

- \rightarrow Even with factorization: 9 rules, 21 premises.
- \rightarrow With pretty-big-step: 6 rules, 7 premises.

Pretty-big-step semantics

Source language

Grammar of λ -terms

$$egin{array}{lll} v &:=& \operatorname{int} n \mid \operatorname{abs} x \, t \ t &:=& \operatorname{val} v \mid \operatorname{var} x \mid \operatorname{app} t \, t \end{array}$$

Call-by-value big-step semantics $(t \Rightarrow v)$

$$\frac{t_1 \Rightarrow \mathsf{abs}\,x\,t \qquad t_2 \Rightarrow v \qquad [x \to v]\,t \Rightarrow v'}{\mathsf{app}\,t_1\,t_2 \Rightarrow v'}$$

Towards pretty-big-step rules

A first attempt:

$$\begin{array}{cccc} \underline{t_1 \Rightarrow v_1 & \operatorname{app} v_1 t_2 \Rightarrow v'} & \underline{t_2 \Rightarrow v_2 & \operatorname{app} v_1 v_2 \Rightarrow v'} \\ & \underline{app \, t_1 \, t_2 \Rightarrow v'} & \underline{app \, v_1 \, t_2 \Rightarrow v'} \\ & \underline{[x \rightarrow v] \, t \, \Rightarrow v'} \\ & \underline{app \, (\operatorname{abs} x \, t) \, v \, \Rightarrow v'} \end{array}$$

→ Similar idea in Cousot and Cousot's bi-inductive semantics (2007)

Intermediate terms

To prevent overlap between the rules, we use intermediate terms:

$$e := \operatorname{trm} t \mid \operatorname{app1} v t \mid \operatorname{app2} v v$$

Definition of the judgment $e \downarrow v$, with trm implicit:

$$\frac{t_1 \Downarrow v_1 \quad \text{app1} \, v_1 \, t_2 \Downarrow v'}{\text{app} \, t_1 \, t_2 \Downarrow v'}$$

$$\frac{t_2 \Downarrow v_2 \quad \text{app2} \, v_1 \, v_2 \Downarrow v'}{\text{app1} \, v_1 \, t_2 \Downarrow v'} \quad \frac{[x \to v] \, t \Downarrow v'}{\text{app2} \, (\text{abs} \, x \, t) \, v \Downarrow v'}$$

Adding exceptions

Value-carrying exceptions and exception handlers

$$t := \ldots \mid \mathsf{raise}\,t \mid \mathsf{try}\,t\,t$$

Two behaviors: return a value or throw an exception carrying a value

$$e \Downarrow b$$
 $b := ret v \mid exn v$

Updated grammar for intermediate terms

$$e := \operatorname{trm} t \mid \operatorname{app1} b t \mid \operatorname{app2} v b \mid \operatorname{raise1} b \mid \operatorname{try1} b t$$

Adding exceptions

Evaluation rules for applications

$$\frac{t_1 \ \Downarrow \ b_1 \qquad \mathsf{app1} \ b_1 \ t_2 \ \Downarrow \ b}{\mathsf{app} \ t_1 \ t_2 \ \Downarrow \ b}$$

$$\frac{t_2 \ \Downarrow \ b_2 \qquad \mathsf{app2} \ v_1 \ b_2 \ \Downarrow \ b}{\mathsf{app1} \ (\mathsf{exn} \ v) \ t_2 \ \Downarrow \ \mathsf{exn} \ v}$$

Adding exceptions

Evaluation rules for applications

$$\frac{t_1 \ \Downarrow \ b_1 \qquad \mathsf{app1} \ b_1 \ t_2 \ \Downarrow \ b}{\mathsf{app} \ t_1 \ t_2 \ \Downarrow \ b}$$

$$\frac{t_2 \ \Downarrow \ b_2 \qquad \mathsf{app2} \ v_1 \ b_2 \ \Downarrow \ b}{\mathsf{app1} \ (\mathsf{exn} \ v) \ t_2 \ \Downarrow \ \mathsf{exn} \ v}$$

Evaluation rules for exception handlers

$$\frac{t_1 \, \Downarrow \, b_1 \, \operatorname{tryl} \, b_1 \, t_2 \, \Downarrow \, b}{\operatorname{try} \, t_1 \, t_2 \, \Downarrow \, b} \qquad \frac{}{\operatorname{tryl} \, (\operatorname{ret} v) \, t \, \Downarrow \, \operatorname{ret} v} \qquad \frac{\operatorname{\mathsf{app}} \, t \, v \, \Downarrow \, b}{\operatorname{\mathsf{tryl}} \, (\operatorname{\mathsf{exn}} v) \, t \, \Downarrow \, b}$$

Adding divergence

Grammars:

```
\begin{array}{lll} b & := & \operatorname{ret} v \ | \ \operatorname{exn} v \\ o & := & \operatorname{ter} b \ | \ \operatorname{div} \\ e & := & \operatorname{trm} t \ | \ \operatorname{app1} ot \ | \ \operatorname{app2} vo \ | \ \operatorname{raise1} o \ | \ \operatorname{try1} ot \end{array}
```

Two judgments defined by a same set of rules:

$$e \Downarrow o$$
 $e \Downarrow^{\mathsf{co}} o$

Adding divergence

Grammars:

```
\begin{array}{lll} b & := & \operatorname{ret} v \ | \ \operatorname{exn} v \\ o & := & \operatorname{ter} b \ | \ \operatorname{div} \\ e & := & \operatorname{trm} t \ | \ \operatorname{app1} ot \ | \ \operatorname{app2} vo \ | \ \operatorname{raise1} o \ | \ \operatorname{try1} ot \end{array}
```

Two judgments defined by a same set of rules:

$$e \Downarrow o$$
 $e \Downarrow^{co} o$

Theorem (equivalence with big-step)

$$t \Downarrow terb \qquad \Leftrightarrow \qquad t \Rightarrow b$$

$$t \Downarrow^{co} \textit{div} \qquad \Leftrightarrow \qquad t \Rightarrow^{\infty}$$

Example pretty-big-step rules

$$\frac{t_1 \Downarrow o_1 \quad \mathsf{app1} \, o_1 \, t_2 \Downarrow o}{\mathsf{app} \, t_1 \, t_2 \Downarrow o} \qquad \frac{t_2 \Downarrow o_2 \quad \mathsf{app2} \, v_1 \, o_2 \Downarrow o}{\mathsf{app1} \, (\mathsf{ter} \, (\mathsf{ret} \, v_1)) \, t_2 \Downarrow o}$$

$$\frac{\mathsf{app1} \, (\mathsf{ter} \, (\mathsf{exn} \, v)) \, t \, \Downarrow \, \mathsf{ter} \, (\mathsf{exn} \, v)}{\mathsf{app1} \, \mathsf{div} \, t \, \Downarrow \, \mathsf{div}}$$

Example pretty-big-step rules

$$\frac{t_1 \, \Downarrow \, o_1 \quad \mathsf{app1} \, o_1 \, t_2 \, \Downarrow \, o}{\mathsf{app} \, t_1 \, t_2 \, \Downarrow \, o} \qquad \frac{t_2 \, \Downarrow \, o_2 \quad \mathsf{app2} \, v_1 \, o_2 \, \Downarrow \, o}{\mathsf{app1} \, (\mathsf{ter} \, (\mathsf{ret} \, v_1)) \, t_2 \, \Downarrow \, o} \\ \\ \frac{\mathsf{app1} \, (\mathsf{ter} \, (\mathsf{exn} \, v)) \, t \, \Downarrow \, \mathsf{ter} \, (\mathsf{exn} \, v)}{\mathsf{app1} \, (\mathsf{ter} \, (\mathsf{exn} \, v)) \, t \, \Downarrow \, \mathsf{ter} \, (\mathsf{exn} \, v)}$$

Factorization of the rules propagating exceptions and divergence:

$$\frac{\mathsf{abort}\,o}{\mathsf{app1}\,o\,t\,\Downarrow\,o}\qquad \text{where} \qquad \frac{\mathsf{abort}\,(\mathsf{ter}\,(\mathsf{exn}\,v))}{\mathsf{abort}\,\mathsf{div}}$$

All pretty-big-step rules

Evaluation rules, where val, ret and ter are implicit.

$$\frac{t_1 \Downarrow o_1 \quad \operatorname{app1} o_1 t_2 \Downarrow o}{\operatorname{app} t_1 t_2 \Downarrow o} \quad \frac{\operatorname{abort} o}{\operatorname{app1} o t \Downarrow o}$$

$$\frac{t_2 \Downarrow o_2 \quad \operatorname{app2} v_1 o_2 \Downarrow o}{\operatorname{app1} v_1 t_2 \Downarrow o} \quad \frac{\operatorname{abort} o}{\operatorname{app2} v o \Downarrow o} \quad \frac{[x \to v] t \Downarrow o}{\operatorname{app2} (\operatorname{abs} x t) v \Downarrow o}$$

$$\frac{t \Downarrow o_1 \quad \operatorname{raisel} o_1 \Downarrow o}{\operatorname{raise} t \Downarrow o} \quad \frac{\operatorname{abort} o}{\operatorname{raisel} o \Downarrow o} \quad \frac{\operatorname{abort} o}{\operatorname{raisel} v \Downarrow \operatorname{exn} v}$$

$$\frac{t_1 \Downarrow o_1 \quad \operatorname{try1} o_1 t_2 \Downarrow o}{\operatorname{try1} t_2 \Downarrow o} \quad \frac{\operatorname{app} t v \Downarrow o}{\operatorname{try1} (\operatorname{exn} v) t \Downarrow o}$$

Pretty-big-step: scaling up to real languages

Side-effects

Generalization of terminating outcomes to carry a memory store:

$$o := \operatorname{ter} m b \mid \operatorname{div}$$

Evaluation judgment in the form $e_{/m} \Downarrow o$. Example rules:

$$\frac{t_{1\,/m_{1}} \Downarrow o_{1} \quad \mathsf{app1}\, o_{1}\, t_{2\,/m_{1}} \Downarrow o}{\mathsf{app}\, t_{1}\, t_{2\,/m_{1}} \Downarrow o} \qquad \frac{t_{2\,/m_{2}} \Downarrow o_{2} \quad \mathsf{app2}\, v_{1}\, o_{2\,/m_{2}} \Downarrow o}{\mathsf{app1}\, (\mathsf{ter}\, m_{2}\, v_{1})\, t_{2\,/m_{1}} \Downarrow o}$$

app $1 \operatorname{div} t_{2/m_1} \Downarrow \operatorname{div}$

Pretty-big-step semantics for loops

Intermediate terms: "for $_i o t_1 t_2 t_3$ ", where $i \in \{1, 2, 3\}$.

Evaluation rules, with the judgment $e_{/m} \downarrow o$.

$$\frac{t_{1\,/m} \Downarrow o_{1} \quad \text{for}_{1} \, o_{1} \, t_{1} \, t_{2} \, t_{3\,/m} \Downarrow o}{\text{for}_{1} \, t_{2} \, t_{3\,/m} \Downarrow o} \quad \frac{}{\text{for}_{1} \, t_{2} \, t_{3\,/m} \Downarrow o} \quad \frac{t_{3\,/m} \Downarrow o_{3} \quad \text{for}_{2} \, o_{3} \, t_{1} \, t_{2} \, t_{3\,/m} \Downarrow o}{\text{for}_{1} \, (\text{ter} \, m \, \text{true}) \, t_{1} \, t_{2} \, t_{3\,/m} \Downarrow o} \quad \frac{t_{2\,/m} \Downarrow o_{2} \quad \text{for}_{3} \, o_{2} \, t_{1} \, t_{2} \, t_{3\,/m} \Downarrow o}{\text{for}_{2} \, (\text{ter} \, m \, tt) \, t_{1} \, t_{2} \, t_{3\,/m} \Downarrow o} \quad \frac{t_{2\,/m} \Downarrow o_{2} \quad \text{for}_{3} \, o_{2} \, t_{1} \, t_{2} \, t_{3\,/m} \Downarrow o}{\text{for}_{2} \, (\text{ter} \, m \, tt) \, t_{1} \, t_{2} \, t_{3\,/m} \Downarrow o} \quad \frac{d}{d} \quad \frac{d}{d}$$

ightarrow From 9 rules and 21 premises to 6 rules with 7 evaluation premises.

Pretty-big-step semantics for core-Caml

Formalization in Coq of a large subset of Caml:

booleans, integers, tuples, algebraic data types, mutable records, boolean operators (lazy and, lazy or, negation), integer operators (negation, addition, subtraction, multiplication, division), comparison operator, functions, recursive functions, applications, sequences, let-bindings, conditionals (with optional else branch), for loops and while loops, pattern matching (with nested patterns, as patterns, or patterns, and when clauses), raise construct, try-with construct with pattern matching, and assertions.

	rules	premises	tokens
Big-step without divergence	71	83	1540
Big-step with divergence	113	143	2263
Pretty-big-step	70	60	1361

- \rightarrow Pretty-big-step reduces the size of the definition by 40%.
- \rightarrow Pretty-big-step reduces the number of premises by 60%.

Pretty-big-step semantics for JavaScript

Formalization in Coq of a large subset of JavaScript (ECMA5): variable declarations, function declarations, function calls, objects, getters, setters, new, delete, access, assignment, unary and binary operators, sequence, conditional, while loop, with construct, this construct, throw, try-catch-finally, return, break, continue, type conversions, primitive functions on objects. Not yet covered:

parsing, switch, arrays, for loops, library functions such as regexps.

	Language	Meta	Total	
	constructs	operations		
Intermediate terms	97	165	262	
Evaluation rules	147	258	432	

Conclusion

In this talk:

- Ouplication associated with big-step semantics
- From big-step to pretty-big-step semantics
- Scaling up to real languages

Additional results described in the paper:

- Type soundness proofs in pretty-big-step
- Pretty-big-step semantics with traces

Conclusion

In this talk:

- Duplication associated with big-step semantics
- From big-step to pretty-big-step semantics
- Scaling up to real languages

Additional results described in the paper:

- Type soundness proofs in pretty-big-step
- Pretty-big-step semantics with traces

Remaining challenges for pretty-big-step:

- Unified proofs for terminating and diverging terms
- Support for arbitrary goto instructions
- Support for concurrency and weak memory models

Thanks!