

Pretty-big-step semantics

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Motivation

Formalization of JavaScript

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Previous work:

- ▶ Semi-formal small-step semantics for the entire language (jssec.net)
- ▶ Informal big-step semantics for the core language (POPL'12)

Current work:

- ▶ Formal big-step semantics for the entire language
- ▶ Interpreter proved correct w.r.t. the semantics

Motivation for big-step

Big-step semantics:

- ▶ more faithful to the reference manual
- ▶ easier than small-step for proving an interpreter
- ▶ easier than small-step for proving a program logic

Small-step semantics considered better-suited for:

- ▶ machine-code semantics → not the case of JS
- ▶ type soundness proofs → no types in JS
- ▶ concurrent languages → no concurrency in JS

Big-step semantics for loops

Semantics of a C-style loop “for (; t_1 ; t_2) { t_3 }”, written “for $t_1 t_2 t_3$ ”,
in terms of the evaluation judgment $t/m \Rightarrow v/m'$.

$$\frac{t_1/m_1 \Rightarrow \text{false}/m_2}{\text{for } t_1 t_2 t_3/m_1 \Rightarrow tt/m_2}$$

$$\frac{t_1/m_1 \Rightarrow \text{true}/m_2 \quad t_3/m_2 \Rightarrow tt/m_3 \quad t_2/m_3 \Rightarrow tt/m_4 \quad \text{for } t_1 t_2 t_3/m_4 \Rightarrow tt/m_5}{\text{for } t_1 t_2 t_3/m_1 \Rightarrow tt/m_5}$$

Big-step semantics for loops: exceptions

Exceptions in terms of the judgment $t/m \Rightarrow^{\text{exn}}/m'$.

$$\frac{t_1/m_1 \Rightarrow^{\text{exn}}/m_2}{\text{for } t_1 t_2 t_3/m_1 \Rightarrow^{\text{exn}}/m_2}$$

$$\frac{t_1/m_1 \Rightarrow \text{true}/m_2 \quad t_3/m_2 \Rightarrow^{\text{exn}}/m_3}{\text{for } t_1 t_2 t_3/m_1 \Rightarrow^{\text{exn}}/m_3}$$

$$\frac{t_1/m_1 \Rightarrow \text{true}/m_2 \quad t_3/m_2 \Rightarrow tt/m_3 \quad t_2/m_3 \Rightarrow^{\text{exn}}/m_4}{\text{for } t_1 t_2 t_3/m_1 \Rightarrow^{\text{exn}}/m_4}$$

$$\frac{t_1/m_1 \Rightarrow \text{true}/m_2 \quad t_3/m_2 \Rightarrow tt/m_3 \quad t_2/m_3 \Rightarrow tt/m_4 \quad \text{for } t_1 t_2 t_3/m_4 \Rightarrow^{\text{exn}}/m_5}{\text{for } t_1 t_2 t_3/m_1 \Rightarrow^{\text{exn}}/m_5}$$

Big-step semantics for loops: divergence

Divergence in terms of the coinductive judgment $t/m \Rightarrow^\infty$ (Leroy 2006).

$$\frac{t_1/m_1 \Rightarrow^\infty}{\text{for } t_1 t_2 t_3/m_1 \Rightarrow^\infty}$$

$$\frac{t_1/m_1 \Rightarrow \text{true}/m_2 \quad t_3/m_2 \Rightarrow^\infty}{\text{for } t_1 t_2 t_3/m_1 \Rightarrow^\infty}$$

$$\frac{t_1/m_1 \Rightarrow \text{true}/m_2 \quad t_3/m_2 \Rightarrow tt/m_3 \quad t_2/m_3 \Rightarrow^\infty}{\text{for } t_1 t_2 t_3/m_1 \Rightarrow^\infty}$$

$$\frac{t_1/m_1 \Rightarrow \text{true}/m_2 \quad t_3/m_2 \Rightarrow tt/m_3 \quad t_2/m_3 \Rightarrow tt/m_4 \quad \text{for } t_1 t_2 t_3/m_4 \Rightarrow^\infty}{\text{for } t_1 t_2 t_3/m_1 \Rightarrow^\infty}$$

Big-step semantics for loops: summary

$$\frac{t_1/m_1 \Rightarrow \text{false}/m_2}{\text{for } t_1 \ t_2 \ t_3/m_1 \Rightarrow t/m_2}$$

$$\frac{t_1/m_1 \Rightarrow \text{true}/m_2 \quad t_3/m_2 \Rightarrow t/m_3 \quad t_2/m_3 \Rightarrow t/m_4 \quad \text{for } t_1 \ t_2 \ t_3/m_4 \Rightarrow t/m_5}{\text{for } t_1 \ t_2 \ t_3/m_1 \Rightarrow t/m_5}$$

$$\frac{t_1/m_1 \Rightarrow^{\text{exn}}/m_2}{\text{for } t_1 \ t_2 \ t_3/m_1 \Rightarrow^{\text{exn}}/m_2}$$

$$\frac{t_1/m_1 \Rightarrow^\infty}{\text{for } t_1 \ t_2 \ t_3/m_1 \Rightarrow^\infty}$$

$$\frac{t_1/m_1 \Rightarrow \text{true}/m_2 \quad t_3/m_2 \Rightarrow^{\text{exn}}/m_3}{\text{for } t_1 \ t_2 \ t_3/m_1 \Rightarrow^{\text{exn}}/m_3}$$

$$\frac{t_1/m_1 \Rightarrow \text{true}/m_2 \quad t_3/m_2 \Rightarrow^\infty}{\text{for } t_1 \ t_2 \ t_3/m_1 \Rightarrow^\infty}$$

$$\frac{t_1/m_1 \Rightarrow \text{true}/m_2 \quad t_3/m_2 \Rightarrow t/m_3 \quad t_2/m_3 \Rightarrow^{\text{exn}}/m_4}{\text{for } t_1 \ t_2 \ t_3/m_1 \Rightarrow^{\text{exn}}/m_4}$$

$$\frac{t_1/m_1 \Rightarrow \text{true}/m_2 \quad t_3/m_2 \Rightarrow t/m_3 \quad t_2/m_3 \Rightarrow^\infty}{\text{for } t_1 \ t_2 \ t_3/m_1 \Rightarrow^\infty}$$

$$\frac{t_1/m_1 \Rightarrow \text{true}/m_2 \quad t_3/m_2 \Rightarrow t/m_3 \quad t_2/m_3 \Rightarrow t/m_4 \quad \text{for } t_1 \ t_2 \ t_3/m_4 \Rightarrow^{\text{exn}}/m_5}{\text{for } t_1 \ t_2 \ t_3/m_1 \Rightarrow^{\text{exn}}/m_5}$$

$$\frac{t_1/m_1 \Rightarrow \text{true}/m_2 \quad t_3/m_2 \Rightarrow t/m_3 \quad t_2/m_3 \Rightarrow t/m_4 \quad \text{for } t_1 \ t_2 \ t_3/m_4 \Rightarrow^\infty}{\text{for } t_1 \ t_2 \ t_3/m_1 \Rightarrow^\infty}$$

→ Even with factorization: 9 rules, 21 premises.

→ With pretty-big-step: 6 rules, 7 premises.

In this talk

Pretty-big-step semantics:

- ▶ construction
- ▶ extension to traces
- ▶ application to core-Caml
- ▶ ~~type soundness proofs~~

Pretty-big-step

Big-step semantics

Grammar of λ -terms

$$\begin{aligned}v &:= \text{int } n \mid \text{abs } x t \\t &:= \text{val } v \mid \text{var } x \mid \text{app } t t\end{aligned}$$

Call-by-value big-step semantics ($t \Rightarrow v$)

$$\frac{}{v \Rightarrow v} \quad \frac{t_1 \Rightarrow \text{abs } x t \quad t_2 \Rightarrow v \quad [x \rightarrow v] t \Rightarrow v'}{\text{app } t_1 t_2 \Rightarrow v'}$$

A first attempt

Big-step rule for applications:

$$\frac{t_1 \Rightarrow \text{abs } x t \quad t_2 \Rightarrow v \quad [x \rightarrow v] t \Rightarrow v'}{\text{app } t_1 t_2 \Rightarrow v'}$$

A first attempt at pretty-big-step rules:

$$\frac{t_1 \Rightarrow v_1 \quad \text{app } v_1 t_2 \Rightarrow v'}{\text{app } t_1 t_2 \Rightarrow v'} \quad \frac{t_2 \Rightarrow v_2 \quad \text{app } v_1 v_2 \Rightarrow v'}{\text{app } v_1 t_2 \Rightarrow v'} \quad \frac{[x \rightarrow v] t \Rightarrow v'}{\text{app } (\text{abs } x t) v \Rightarrow v'}$$

→ Similar idea in Cousot and Cousot's bi-inductive semantics (2007)

Intermediate terms

To prevent overlap between the rules, we use intermediate terms.

$$e := \text{trm } t \mid \text{app1 } vt \mid \text{app2 } vv$$

Definition of the judgment $e \Downarrow v$, with trm implicit

$$\frac{}{v \Downarrow v} \qquad \frac{t_1 \Downarrow v_1 \quad \text{app1 } v_1 t_2 \Downarrow v'}{\text{app } t_1 t_2 \Downarrow v'}$$
$$\frac{t_2 \Downarrow v_2 \quad \text{app2 } v_1 v_2 \Downarrow v'}{\text{app1 } v_1 t_2 \Downarrow v'} \qquad \frac{[x \rightarrow v] t \Downarrow v'}{\text{app2 } (\text{abs } xt) v \Downarrow v'}$$

Adding exceptions

Value-carrying exceptions and exception handlers

$$t := \dots \mid \text{raise } t \mid \text{try } t t$$

Two behaviors: return a value or throw an exception.

$$e \Downarrow b \qquad b := \text{ret } v \mid \text{exn } v$$

Generalization of intermediate terms

Need to generalize intermediate terms

$$\frac{t_1 \Downarrow b_1 \quad \text{app1 } b_1 t_2 \Downarrow b}{\text{app } t_1 t_2 \Downarrow b}$$

$$\frac{}{\text{app1 (exn } v) t_2 \Downarrow \text{exn } v} \quad \frac{t_2 \Downarrow b_2 \quad \text{app2 } v_1 b_2 \Downarrow b}{\text{app1 (ret } v_1) t_2 \Downarrow b}$$

New grammar of intermediate terms

$$e ::= \text{trm } t \mid \text{app1 } b t \mid \text{app2 } v b \mid \text{raise1 } b \mid \text{try1 } b t$$

Pretty-big-step rules for exceptions

$$\frac{}{v \Downarrow v} \qquad \frac{t_1 \Downarrow b_1 \quad \text{app1 } b_1 t_2 \Downarrow b}{\text{app } t_1 t_2 \Downarrow b} \qquad \frac{}{\text{app1 } (\text{exn } v) t \Downarrow \text{exn } v}$$

$$\frac{t_2 \Downarrow b_2 \quad \text{app2 } v_1 b_2 \Downarrow b}{\text{app1 } v_1 t_2 \Downarrow b} \qquad \frac{}{\text{app2 } v (\text{exn } v) \Downarrow \text{exn } v}$$

$$\frac{[x \rightarrow v] t \Downarrow b}{\text{app2 } (\text{abs } x t) v \Downarrow b}$$

$$\frac{t_1 \Downarrow b_1 \quad \text{try1 } b_1 t_2 \Downarrow b}{\text{try } t_1 t_2 \Downarrow b} \qquad \frac{}{\text{try1 } v t \Downarrow v} \qquad \frac{\text{app } t v \Downarrow b}{\text{try1 } (\text{exn } v) t \Downarrow b}$$

$$\frac{t \Downarrow b_1 \quad \text{raise1 } b_1 \Downarrow b}{\text{raise } t \Downarrow b} \qquad \frac{}{\text{raise1 } v \Downarrow \text{exn } v} \qquad \frac{}{\text{raise1 } (\text{exn } v) \Downarrow \text{exn } v}$$

Adding divergence

Outcome of an evaluation: termination or divergence

$$o := \text{ter } b \mid \text{div}$$

New grammar of intermediate terms

$$e := \text{trm } t \mid \text{app1 } o t \mid \text{app2 } v o \mid \text{raise1 } o \mid \text{try1 } o t$$

Evaluation rules

$$\frac{t_1 \Downarrow o_1 \quad \text{app1 } o_1 t_2 \Downarrow b}{\text{app } t_1 t_2 \Downarrow b} \qquad \frac{}{\text{app1 div } t \Downarrow \text{div}}$$

The abort predicate

We want to factorize pairs of similar rules, such as:

$$\frac{}{\text{app1 (exn } v) t \Downarrow (\text{exn } v)}$$

$$\frac{}{\text{app1 div } t \Downarrow \text{div}}$$

Solution:

$$\frac{\text{abort } o}{\text{app1 } o t \Downarrow o}$$

where “abort” characterizes exceptions (exn v) and divergence (div).

Summary: grammars and judgments

Grammars:

$$b := \text{ret } v \mid \text{exn } v$$
$$o := \text{ter } b \mid \text{div}$$
$$e := \text{trm } t \mid \text{app1 } o t \mid \text{app2 } v o \mid \text{raise1 } o \mid \text{try1 } o t$$

Judgments:

$$\text{abort } o$$
$$e \Downarrow o$$
$$e \Downarrow^{\text{co}} o$$

Theorem (equivalence with big-step)

$$t \Downarrow \text{ter } b \quad \Leftrightarrow \quad t \Rightarrow b$$
$$t \Downarrow^{\text{co}} \text{div} \quad \Leftrightarrow \quad t \Rightarrow^{\infty}$$

Summary: rules

$$\frac{}{v \Downarrow v} \quad \frac{t_1 \Downarrow o_1 \quad \text{app1 } o_1 t_2 \Downarrow o}{\text{app } t_1 t_2 \Downarrow o}$$

$$\frac{\text{abort } o}{\text{app1 } o t \Downarrow o}$$

$$\frac{t_2 \Downarrow o_2 \quad \text{app2 } v_1 o_2 \Downarrow o}{\text{app1 } v_1 t_2 \Downarrow o}$$

$$\frac{\text{abort } o}{\text{app2 } v o \Downarrow o}$$

$$\frac{[x \rightarrow v] t \Downarrow o}{\text{app2 } (\text{abs } x t) v \Downarrow o}$$

$$\frac{t \Downarrow o_1 \quad \text{raise1 } o_1 \Downarrow o}{\text{raise } t \Downarrow o}$$

$$\frac{\text{abort } o}{\text{raise1 } o \Downarrow o}$$

$$\frac{}{\text{raise1 } v \Downarrow \text{exn } v}$$

$$\frac{t_1 \Downarrow o_1 \quad \text{try1 } o_1 t_2 \Downarrow o}{\text{try } t_1 t_2 \Downarrow o}$$

$$\frac{}{\text{try1 } v t \Downarrow v}$$

$$\frac{\text{app } t v \Downarrow o}{\text{try1 } (\text{exn } v) t \Downarrow o}$$

$$\frac{}{\text{try1 div } t \Downarrow \text{div}}$$

Traces

Definition of traces

A trace records I/O interactions and ϵ -transitions.

A terminating program has a finite trace.

A diverging program has an infinite trace.

$$\alpha := \epsilon \mid \text{in } n \mid \text{out } n$$
$$\tau := \text{list } \alpha$$
$$\sigma := \text{stream } \alpha$$
$$o := \text{ter } \tau b \mid \text{div } \sigma$$

→ We are not using possibly-infinite traces (coinductive lists) like Nakata and Uustalu (2009) and Danielsson (2012).

Operation on traces

Concatenation of a finite trace τ to the front

$$\tau \cdot \tau' \qquad \tau \cdot \sigma \qquad \tau \cdot o$$

Equivalence of two traces up to finite consecutive insertions of ϵ -transitions

$$\frac{o \approx o'}{\epsilon^n \cdot [\alpha] \cdot o \approx \epsilon^m \cdot [\alpha] \cdot o'}$$

Trace semantics in pretty-big-step

Every rule appends an ϵ -transition in order to be *productive*.

$$\frac{}{v \Downarrow \text{ter } [\epsilon] v} \quad \frac{t_1 \Downarrow o_1 \quad \text{app1 } o_1 t_2 \Downarrow o}{\text{app } t_1 t_2 \Downarrow [\epsilon] \cdot o} \quad \frac{\text{abort } o}{\text{app1 } o t \Downarrow o}$$
$$\frac{t_2 \Downarrow o_2 \quad \text{app2 } v_1 o_2 \Downarrow o}{\text{app1 } (\text{ter } \tau v_1) t_2 \Downarrow \tau \cdot o} \quad \frac{\text{abort } o}{\text{app2 } v o \Downarrow o}$$
$$\frac{[x \rightarrow v] t \Downarrow o}{\text{app2 } (\text{abs } x t) (\text{ter } \tau v) \Downarrow \tau \cdot o}$$

Trace semantics in pretty-big-step, cont.

I/O operations are recorded in the trace.

$$\frac{t \Downarrow o_1 \quad \text{write1 } o_1 \Downarrow o}{\text{write } t \Downarrow [\epsilon] \cdot o}$$

$$\frac{}{\text{write1 } (\text{ter } \tau n) \Downarrow \text{ter } \tau \cdot [\text{out } n] tt}$$

$$\frac{t \Downarrow o_1 \quad \text{read1 } o_1 \Downarrow o}{\text{read } t \Downarrow [\epsilon] \cdot o}$$

$$\frac{}{\text{read1 } (\text{ter } \tau tt) \Downarrow \text{ter } \tau \cdot [\text{in } n] n}$$

$$\frac{\text{abort } o}{\text{read1 } o \Downarrow o}$$

$$\frac{\text{abort } o}{\text{write1 } o \Downarrow o}$$

Benefits of trace semantics

Theorem (finite traces can only be produced by finite derivations)

$$e \Downarrow^{\text{co}} \text{ter} \tau v \quad \Leftrightarrow \quad e \Downarrow \text{ter} \tau v$$

So, we do not need the inductive judgment: the coinductive one suffices.

Theorem (equivalence with big-step)

$$t \Downarrow^{\text{co}} \text{ter} \tau b \quad \Leftrightarrow \quad t \Rightarrow b / \tau$$

$$t \Downarrow^{\text{co}} \text{div} \sigma \quad \Leftrightarrow \quad t \Rightarrow^{\infty} / \sigma$$

Proofs with trace semantics: problems

A typical simulation theorem

$$\llbracket e \rrbracket \Downarrow^{\text{co}} o \quad \rightarrow \quad \exists o'. \quad o' \approx o \quad \wedge \quad e \Downarrow^{\text{co}} o'$$

Coinductive proof? No luck!

1. \exists is not coinductive
2. \wedge is not coinductive
3. o' is not coinductive

→ Yet, coinductive reasoning is morally correct.

Pretty-big-step: scaling up

Scaling up to real languages

What's next:

- ▶ the generic abort rule
- ▶ semantics of side-effects
- ▶ semantics of loops
- ▶ formalization of core-Caml

Abort rules

Many similar abort rules: can we factorize them?

$$\frac{\text{abort } o}{\text{app1 } o t \Downarrow o}$$

$$\frac{\text{abort } o}{\text{app2 } v o \Downarrow o}$$

$$\frac{\text{abort } o}{\text{raise1 } o \Downarrow o}$$

The generic abort rule

The auxiliary function “getout”

$$\begin{aligned} \text{getout } (\text{app1 } o t) &\equiv \text{Some } o \\ \text{getout } (\text{app2 } v o) &\equiv \text{Some } o \\ \text{getout } (\text{raise1 } o) &\equiv \text{Some } o \end{aligned}$$
$$\begin{aligned} \text{getout } (\text{trm } t) &\equiv \text{None} \\ \text{getout } (\text{try1 } o t) &\equiv \text{None} \end{aligned}$$

The generic abort rule, which replaces the rules from the previous slide

$$\frac{\text{getout } e = \text{Some } o \quad \text{abort } o}{e \Downarrow o}$$

Side-effects

Generalization of terminating outcomes to carry a memory store:

$$o := \text{term } b \mid \text{div}$$

Evaluation judgment in the form $e /m \Downarrow o$. Example rules:

$$\frac{t_1 /m \Downarrow o_1 \quad \text{app1 } o_1 t_2 /m \Downarrow o}{\text{app } t_1 t_2 /m \Downarrow o}$$

$$\frac{t_2 /m' \Downarrow o_2 \quad \text{app2 } v_1 o_2 /m' \Downarrow o}{\text{app1 } (\text{term}' v_1) t_2 /m \Downarrow o}$$

Pretty-big-step semantics for loops

A single intermediate term “for i o t_1 t_2 t_3 ”, where $i \in \{1, 2, 3\}$.

Evaluation rules, with the judgment $e /_m \Downarrow o$.

$$\frac{t_1 /_m \Downarrow o_1 \quad \text{for } 1 \ o_1 \ t_1 \ t_2 \ t_3 /_m \Downarrow o}{\text{for } t_1 \ t_2 \ t_3 /_m \Downarrow o}$$

$$\overline{\text{for } 1 \ (\text{ret } m \ \text{false}) \ t_1 \ t_2 \ t_3 /_{m'} \Downarrow \text{ret } m \ tt}$$

$$\frac{t_3 /_m \Downarrow o_3 \quad \text{for } 2 \ o_3 \ t_1 \ t_2 \ t_3 /_m \Downarrow o}{\text{for } 1 \ (\text{ret } m \ \text{true}) \ t_1 \ t_2 \ t_3 /_{m'} \Downarrow o}$$

$$\frac{t_2 /_m \Downarrow o_2 \quad \text{for } 3 \ o_2 \ t_1 \ t_2 \ t_3 /_m \Downarrow o}{\text{for } 2 \ (\text{ret } m \ tt) \ t_1 \ t_2 \ t_3 /_{m'} \Downarrow o}$$

$$\frac{\text{for } t_1 \ t_2 \ t_3 /_m \Downarrow o}{\text{for } 3 \ (\text{ret } m \ tt) \ t_1 \ t_2 \ t_3 /_{m'} \Downarrow o}$$

$$\frac{\text{abort } o}{\text{for } i \ o \ t_1 \ t_2 \ t_3 /_m \Downarrow o}$$

Pretty-big-step semantics for core-Caml

Formalization of core-Caml:

booleans, integers, tuples, algebraic data types, mutable records, boolean operators (lazy and, lazy or, negation), integer operators (negation, addition, subtraction, multiplication, division), comparison operator, functions, recursive functions, applications, sequences, let-bindings, conditionals (with optional *else* branch), *for* loops and *while* loops, pattern matching (with nested patterns, *as* patterns, *or* patterns, and *when* clauses), *raise* construct, *try-with* construct with pattern matching, and assertions.

| | rules | premises | tokens |
|-----------------------------|-------|----------|--------|
| Big-step without divergence | 71 | 83 | 1540 |
| Big-step with divergence | 113 | 143 | 2263 |
| Pretty-big-step | 70 | 60 | 1361 |

→ Pretty-big-step reduces the size of the definition by 40%.

→ Pretty-big-step reduces the number of premises by more than a factor 2.

Thanks!